11. x^2, $\sqrt[3]{x^6+x^2}$

same

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^6+x^2}}{x^2}$$

$$(x^6)^{\frac{1}{3}} = x^2$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^6+x^2}}{\sqrt[3]{x^6}}$$

$$\lim_{x\to\infty} \sqrt[3]{\frac{x^6+x^2}{x^6}} = \lim_{x\to\infty} \sqrt[3]{1+\frac{x^2}{x^6}}$$

$$= \lim_{x\to\infty} \sqrt[3]{1+\frac{1}{x^4}} = 1$$

$$\frac{1}{x^4} \to 0$$

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33. 3^x $\sqrt{9^x+2^x}$ $\sqrt{9^x-4^x}$

$$\frac{\infty}{\infty} \quad \lim_{x\to\infty} \frac{\sqrt{9^x+2^x}}{3^x}$$

$$3^x = \sqrt{(3^x)^2} = \sqrt{3^{2x}} = \sqrt{(3^2)^x} = \sqrt{9^x}$$

$$\frac{\sqrt{9^x+2^x}}{\sqrt{9^x}}$$

$$\sqrt{\frac{9^x+2^x}{9^x}}$$

$$\sqrt{\left(1+\frac{2^x}{9^x}\right)} = 1 \qquad \left(\frac{2}{9}\right)^x$$

$$\frac{2^x}{9^x} \to 0$$

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8.4 a Improper Integrals

$$\int_1^{\infty} \frac{1}{x^2}\,dx$$ converges to 1

infinity makes it improper

$$\lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx \overset{?}{=} 1$$

$$\frac{x^{-1}}{-1}$$

$$\lim_{b\to\infty} \left.\frac{-1}{x}\right|_1^b = \lim_{b\to\infty}\left(\cancel{\frac{-1}{b}}^{\,0} - \frac{-1}{1}\right) = 1$$

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$$\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{\sqrt{x}}\,dx$$

diverges

$$x^{-\frac{1}{2}}$$

$$\lim_{b\to\infty} \left. 2x^{\frac{1}{2}} \right|_1^b$$

$$\lim_{b\to\infty} \underset{\downarrow \atop \infty}{2\sqrt{b}} - 2\sqrt{1} = \infty$$

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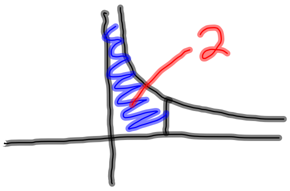
$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$ also improper



$$\lim_{b\to 0^+} \int_b^1 x^{-\frac{1}{2}}\,dx$$

$$\lim_{b\to 0^+} 2x^{\frac{1}{2}} \Big|_b^1$$

$$\lim_{b\to 0^+} 2\sqrt{1} - \overset{\to 0}{2\sqrt{b}} = 2$$

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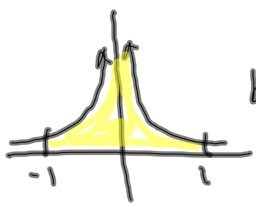
$$\int_{-1}^1 \frac{1}{x^2}\,dx = \int_{-1}^0 \frac{1}{x^2} + \int_0^1 \frac{1}{x^2}\,dx$$



$$\lim_{b\to 0^-} \int_{-1}^b \frac{1}{x^2}\,dx + \lim_{b\to 0^+} \int_b^1 \frac{1}{x^2}\,dx$$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \int_{\infty}^0 + \int_0^{\infty}$$

$$= \lim_{b\to -\infty} \int_b^0 + \lim_{b\to\infty} \int_0^b$$

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